SVZ sum rules : $30 \oplus 1$ years later

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Abstract

For this exceptional 25th anniversary of the QCD-Montpellier series of conferences initiated in 85 with the name "Non-perturbative methods", we take the opportunuity to celebrate the $30 \oplus 1$ years of the discovery of the SVZ (also called ITEP, QCD or QCD spectral) sum rules by M.A. Shifman, A.I. Vainshtein and V.I. Zakahrov in 79 [1]. In this talk, I have the duty to present the status of the method. I shall (can) not enumerate the vast area of successful applications of sum rules in hadron physics but I shall focus on the historical evolution of field and its new developments. More detailed related discussions and more complete references can be found in the textbooks [2, 3].

Keywords: QCD spectral sum rules, Non-perturbative methods.

1. Introduction

This talk, which is supposed to be a status review, aims to complete the contributions of the two inventors (M.A. Shifman and V.I. Zakahrov), the historical talk of H.G. Dosch on baryon sum rules and the talks given by different orators on various modern applications of the sum rules in this session. This ceremonial session is chaired by E. de Rafael who has participated with enthousiasm in the developments of the field.

To my opinion, the SVZ sum rule [1] is one of the most important discovery of the 20th century in high energy physics phenomenology [2–14]. This discovery has been recognized by the award of the Sakurai price to SVZ in 1999.

This year is the 25th annivesary of the QCD Montpellier series of conferences where S and Z participate regularly and where Z belongs as a committee member during several years. Then, it looks a natural recognition of their efforts and works to also celebrate, in the same time, the $30 \oplus 1$ years of their discovery, also because the chairman of this conference works intensively in this field since its invention.

The SVZ ideas were fantastic as they have formulated in QCD, with the inclusion of the non-perturbative condensate contributions, the old idea of (ad hoc) duality [15], superconvergent [16] and smearing [17] phenomenological sum rules used in the era of 60-76. In the same time, some attempts to improve these pre-QCD sum rules using perturbative QCD have been however done [18] using finite energy sum rules (FESR) contour techniques à la Shankar [19] for testing the breaking of the convergence of the Weinberg sum rules [16] when the perturbative current quark masses are switch on. As a consequence, some combinations of superconvergent sum rules for non-zero light quark masses have been proposed.

2. Different forms of the sum rules

From these short historical reminders, the SVZ sum rules are, like previous others, alternative improvements of the well-known Källen-Lehmann dispersion relation, which for a hadronic two-point correlator reads:

$$\begin{aligned}\Pi_H(Q^2) &\equiv i\int d^4x\, e^{iqx}\langle 0|\mathcal{T} J_H(x)J_H^\dagger(0)|0\rangle \\ &= \int_{t<}^{\infty}\frac{dt}{t+Q^2+i\epsilon}\frac{1}{\pi}\mathrm{Im}\Pi(t)+\ldots \qquad (1)\end{aligned}$$

where $Q^2 \equiv -q^2 > 0$, ... means arbitrary subtraction terms which are polynomial in t; $J_H(x)$ is any hadronic current with definite quantum numbers built from quarks and/or gluon fields. This well-known dispersion relation is very important as it relates $\Pi(Q^2)$ which can be calculated in QCD provided that Q^2 is much larger than the QCD scale Λ^2, with its imaginary part which can be measured at low energy from experiments.

2.1. The SVZ sum rules

SVZ improvements act in the two sides of this dispersion relation.

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• *Exponential sum rules*

The *popular sum rule* is obtained when one takes an infinite number of derivatives n of the correlator in Q^2 but keeping the ratio $Q^2/n \equiv M^2 \equiv \tau^{-1}$ (Borel/Laplace sum rule scale) fixed. In this way, one can eliminate the substraction terms and the dispersion becomes an exponential:

$$\mathcal{L}(\tau) = \int_{t_<}^{\infty} dt\, e^{-t\tau}\, \frac{1}{\pi}\mathrm{Im}\Pi(t)\,, \tag{2}$$

where the exponential has the nice feature to enhance the lowest resonance contribution in the spectral integral. Another related sum rule is the ratio of sum rules:

$$\mathcal{R}(\tau) \equiv -\frac{d}{d\tau}\log\mathcal{L}(\tau)\,, \tag{3}$$

which is often used in the literature for extracting the lowest ground state hadron mass as at the optimization point (disussed later on) $\mathcal{R}(\tau_0) \simeq M_R^2$.

• *Moment sum rule*

An alternative sum rule, which possesses the same property of enhancing the lowest resonance contribution is the moment sum rule:

$$\begin{aligned}\mathcal{M}_n &\equiv \frac{(-1)^n}{n!}\frac{d^n}{(dQ^2)^n}\Pi(Q^2)\\ &= \int_{t_<}^{\infty}\frac{dt}{(t+Q^2)^n}\,\frac{1}{\pi}\mathrm{Im}\Pi(t),\end{aligned} \tag{4}$$

which has been (first discussed) to my knowledge by Yndurain using the positivity of the spectral function [22] and has been applied later on to light quarks [2, 23, 24] and to heavy quark systems [1, 10, 11].

2.2. Some other QCD spectral sum rules (QSSR)

Different variants of the SVZ sum rules have been proposed in the literature such as the [1]:

• *Finite energy sum rule (FESR)*

This sum rule [15, 18, 28]:

$$\mathcal{M}_n \equiv \int_{t_<}^{t_c} dt\, t^n\, \frac{1}{\pi}\mathrm{Im}\Pi(t) \quad n > 0\,, \tag{5}$$

can derived from the Laplace/Borel sum rule in the limit $\tau \to 0$. This FESR is often called local duality (in the contrast to the above called global duality) sum rule demonstrates the correlation between the lowest resonance mass and the continuum threshold t_c, where one should note that some choices of t_c in the sum rule literature do not satisfy this constraint. FESR is an useful complement of the previous sum rules.

[1]We shall not discuss the analytic continuation and infinite norm approaches [25, 26], based on a χ^2-minimization fitting procedure in the complex q^2-plane, where some comments on this approach have been given in [2].

• *Gaussian sum rule*

$$G(s,\sigma) = \int_{t_<}^{\infty} dt\, \exp^{-\frac{(t+s)^2}{\sigma}}\, \frac{1}{\pi}\mathrm{Im}\Pi(t) \tag{6}$$

It is centered at s with finite width resolution $\sqrt{4\pi\sigma}$. Mathematically, it can be used to derive the Borel/Laplace and FESR sum rules [28].

• *τ-like sum rule*

It has been originally introduced by [29]:

$$R_{nm}(M_\tau^2) = \int_0^{M_\tau^2} dt\, t^n\left(1-\frac{t}{M_\tau^2}\right)^m \frac{1}{\pi}\mathrm{Im}\Pi(t)\,, \tag{7}$$

where $m = 2$ for the physical $\tau \to \nu_\tau + hadrons$ decay. The threshold factor suppresses the contribution near the real axis and improves the quality of the sum rule. This sum rule has been generalized in the literautre by the introduction of an arbitrary weight factor.

• *ϕ-like and (pseudo)scalar sum rules*

In a similar way, a sum rule which suppresses the leading contribution threshold t_c term has been inspired for extracting the stange quark mass from the ϕ-meson sum rule [30]:

$$R(t_c) = \int_{t_<}^{t_c} dt\,\left(1-2\frac{t}{t_c}\right)\frac{1}{\pi}\mathrm{Im}\Pi(t)\,, \tag{8}$$

while a combination of (pseudo)scalar sum rules of the corresponding two-point correlator $\Psi_{(5)}(Q^2)$not sensitive to the leading perturbative (PT) contribution [31]:

$$\begin{aligned}&\int_{t_<}^{\infty}\frac{dt}{t}\exp^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Psi_{(5)}(t) = \Psi_{(5)}(0)\\ &+\frac{3}{4\pi^2}(\bar{m}_u \pm \bar{m}_s)^2\tau^{-1}\left(\frac{\bar{\alpha}_s}{\pi}\right)+\ldots,\end{aligned} \tag{9}$$

has been introduced for extracting:

$$\Psi_{(5)}(0) \equiv -(m_u \mp m_s)\langle\bar{u}u \mp \bar{s}s\rangle + \text{Pert. terms}, \tag{10}$$

entering the Gell-Mann-Oakes-Renner relation.

• *Double ratios of sum rules (DRSR)*

It is defined as:

$$r_{ij} = \frac{\mathcal{R}_i}{\mathcal{R}_j}\,, \tag{11}$$

for two channels i and j and is useful for extracting with a high accuracy the splittings of mesons due to the vanishing of the α_s PT corrections and non-flavoured terms in the QCD expression of the sum rule [32].

2.3. Parametrizations of the Spectral function

• *Success and test of the naïve duality ansatz*

The spectral function Im $\Pi(t)$ can be measured inclusively from the data like $e^+e^- \to hadrons$, $\tau \to \nu_\tau + hadrons$. In most cases, where hadron masses need to be predicted, there are no data available. Then, it is usual to use the minimal duality ansatz parametrization of the spectral function:

$$\textit{One resonance} \oplus \textit{QCD continuum}\ \theta(t-t_c)\ , \quad (12)$$

where t_c is the continuum threshold and the QCD continuum comes from the discontinuity of the QCD diagrams in the OPE in order to ensure the matching of the two sides of the sum rules at high-energies [2]. The naïve duality anstaz in Eq. (12) works quite well in the different applications of the SVZ sum rules within the corresponding expected 10-20% accuracy of the approach. A test of this model from $e^+e^- \to hadrons$ and charmonium data have been done in [3] and has been quite satisfactory. Another successful test is the parametrization of the continuum of the pion spectral function by 3π final states using ChPT constraints [34], where the prediction for the sum of light quark masses remains the same as the one where a pion $\oplus$ a narrow $\pi(1300)$ is used despite the fact the m_π^2 is relatively light compared to the hadronic scale.

• *Duality violation*

However, if one pushes the accuracy of the approach like, e.g., using τ-decay data [29, 35], one becomes sensitive to the detailed structure of the spectral function and a *duality violation* can affect the analysis. This problem is discussed in details in Shifman's talk [5, 36]. In his talk, de Rafael [39] also discusses from a mathematical view, a large N_c toy model (Von Mangoldt) for the spectral function which also leads to an oscillation behaviour and which is can be related to the previous model. Some phenomenological implications of the model are discussed in [35, 37, 38].

3. The original SVZ - Expansion

The SVZ ideas are not only related to the discovery of sum rules. More important, they have proposed a new way to phenomenologically parametrize (approximately) the non-perturbative (confinement) aspect of QCD beyond perturbation theory using an operator product expansion (OPE) à la Wilson in terms of the vacuum condensates of quark or/and gluons of higher and higher dimensions.

[2]In [33] the continuum threshold t_c is proposed to contain corrections of $O(c_n/t_c^n)$ where c_n are fitted from the data. However, it is not clear that the two sides of the sum rules match at high energy.

In addition to the well-known quark condensate $\langle\bar\psi\psi\rangle$ entering to the Gell-Mann-Oakes-Renner relation, SVZ have advocated the non-zero values of the gluon condensate $\langle\alpha_s G^2\rangle$ where they found to be about 0.04 GeV^4 from charmonium sum rules, which has been verified in lattice simulations (see Zakharov's talk [4]) and from e^+e^- data [28, 40, 41]. More recent and refined analyzes indicate that its value is slightly higher of about (0.06 ± 0.01) GeV^4 [30, 42] which goes in line with the lower bound of about 0.08 GeV^4 obtained in [9, 20] from heavy quark exponential moments. The unconclusive extraction of $\langle\alpha_s G^2\rangle$ from a fit of τ-decay data [43] may be due to the small value of the gluon condensate contribution in this channel where it acquires an extra-α_s correction, to its correlation with the other QCD parameters namely α_s, $\langle\bar\psi\psi\rangle^2$ and an (eventual) quadratic $1/q^2$-term (see Zakharov's talk and next section). Indeed, the gluon condensates play an important rôle in gluodynamics (low-energy theorems, gluonia sum rules,...) and in some bag models as $\langle\alpha_s G^2\rangle$ is directly related to the vacuum energy density. Taking the example of the Adler function in $e^+e^- \to hadrons$, it reads within the SVZ-expansion:

$$-Q^2\frac{d}{dQ^2}\,\Pi(Q^2) = \sum_{d=0,1,2,...}\frac{C_d\langle 0|O_d|0\rangle}{Q^d} \qquad (13)$$

The anatomy of the OPE up to $p = 3$ is :

- $d = 0$: usual PT series ($a_s \equiv \alpha_s/\pi$):
  $C_0 = 1 + a_s + 1.640\ a_s^2 + 6.371\ a_s^3 + 49.076\ a_s^4 +$
  $\Delta_N \equiv \sum_{n>N} c_n\alpha_s^n$ where Δ_N is unknown.
- $d = 2$: $\bar m_q^2$: small mass corrections
- $d = 4$: $\langle\alpha_s\ G^2\rangle$, $m_q\langle\bar\psi_q\psi_q\rangle$:
  gluon and quark condensates
- $d = 6$: $\alpha_s\langle\bar\psi_q\psi_q\rangle^2$, $g f_{abc}\langle G^aG^bG^c\rangle$, $m_q g\langle\bar\psi\lambda^a\sigma_{\mu\nu}G_a^{\mu\nu}\rangle$:
  four-quark, triple gluon and mixed condensates,

where one should notiice that in this channel the coefficient of the triple gluon condensate vanishes in the chiral limit $m_q = 0$.

4. Theoretical progresses

There have been different steps for improving the original SVZ sum rules.

4.1. Optimization criteria from quantum mechanics

The question to ask is how one can extract an optimal information on resonance properties from the sum rules and in the same time the OPE remains convergent. The original SVZ proposal is to find a window where the resonance contribution is bigger than the QCD continuum one and where the non-perurbative terms remain

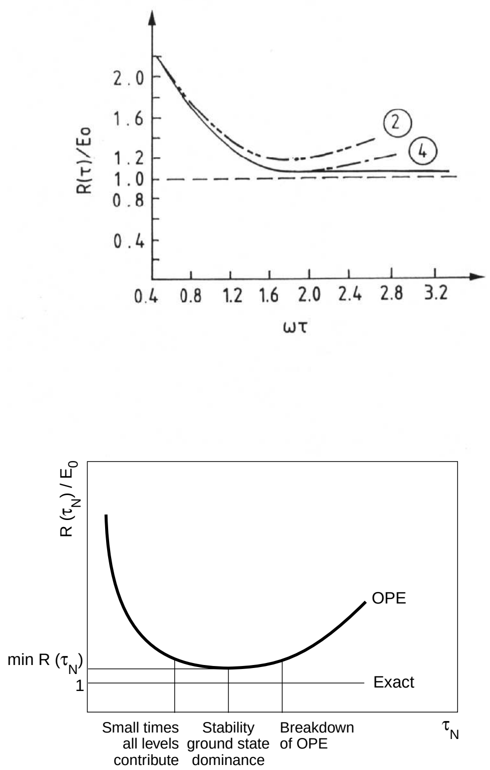

Figure 1: Ratio of moments $\mathcal{R}(\tau)$ in Eq. (3) versus the sum rule variable τ: a) harmonic oscillator. b) charmonium.

reasonnable corrections. Numerically, the argument is handwaving as the percent of contribution to be fixed is arbitrary. In a series of papers, Bell and Bertlmann [9, 20] have investigated this problem using the harmonic oscillator within the exponential moment sum rules. The sum rule variable τ is here an imaginary time variable. The analysis of the ratio of moments $\mathcal{R}(\tau)$ defined in Eq. (3) is shown in Fig. 1a), where one can observe that the exact solution (ground state energy E_0) is reached when more and more terms of the series are added and the optimal information is reached at the minimum of τ for a truncated series. The position of this minimum coïcınde with the SVZ sum rule window but more rigourous. For a comparison, the case of the J/ψ mass is shown in Fig. 1b).

Another free parameter in the phenomenological analysis is the value of the continuum threshold t_c. Many authors *adjust its value* at the intuitive mass of the next radial hadron excitation. This procedure can be false as the QCD continuum only smears all high mass radial excitations, and what is important is the area in the sum rule integral. As the value of t_c is like the sum rule variable an external parameter, one can also require that the physical observables (the lowest resonance parameters) are insensitive to its change. In some cases, this t_c stability is not reached due to the simplest form of the ansatz in Eq. (12). In this case, the complementary use of FESR is useful due to the correlation of t_c with the mass of the lowest resonance (but not with the one of the radial excitation) [28].

4.2. Renormalizations and radiative corrections

SVZ original works have been done to lowest order in α_s. There have been intensive activities for improving the SVZ during the period of 80-90 :

- Inclusion of the PT α_s corrections to the exponential sum rules reveals inverse Laplace transform properties rather than a Borel one [21].
- Mixing of operators under renormalization and evaluation of their anomalous dimensions give a more precise meaning of the condensates where some combinations have been found to be renormalization group invariant [29, 44].
- Absorption of the light quark mass singularities into the condensates leads to the definition of normal or non-normal ordered quark condensate [21, 29, 45].
- Evaluation of the contributions of high-dimension condensates for testing the convergence of the OPE [46].
- Evaluation of the higher order PT corrections [47, 48] and of the ones of the Wilson coefficients of the condensates in some channels [49].

However, despite these large amounts of efforts in the past, it is disappointing to note that most of the recent applications of QSSR only limit to the LO in α_s.

5. Traditional QSSR phenomenology

Since the original work of SVZ [1], the rich conventional phenomenology of QSSR has been reviewed in [2, 3, 6–14]. The different talks given in this session indicate the continuous and wide range of activities in this field. I flash below a panorama of its impressive applications in hadron physics [3]:

- ρ meson, gluon condensate, charm mass since 1979 [1]
- Meson spectroscopy since 1981 [10]
- Light quark masses since 1981 [21, 52]
- Corrections to π and K PCAC since 1981 [3, 31, 53]
- Heavy quark masses since 1979 [1, 54–58]

[3]The approach has been also applied to other QCD-like models like composite models [50] and supersymmetry [51].

- Condensates since 1979 [1, 10, 26, 28, 30, 40–43, 57–59]
- Heavy-light mesons since 1978 [60, 61]
- Light baryons since 1981 [6, 62, 63]
- Heavy baryons since 1992 [64]
- Gluonium since 1981 [65, 66]
- Light hybrids since 1987 [67]
- Heavy hybrids since 1985 [3, 68]
- Four-quarks, molecules since 1985 [69, 70]
- Hadronic decays since 84: Vertex [71, 72], Light Cone [73]
- τ-decay since 1988 [29, 35, 74, 75]
- Thermal and in-medium hadrons since 1986 [76, 77] .

In most applications, the sum rule predictions using the optimization criteria are successful compared with data when available or with some other non-perturbative approaches.

6. Large order and quadratic terms in PT theory

Besides the question of duality violation for the spectral function, it is equally important to control the large order terms of the QCD perturbative series. This issue is discussed in details in the talk of Zakharov [4] which we only sketch here. One attempts to answer the nature of the reminder of the PT series for large N:

$$\Delta_N \equiv \sum_{n>N} c_n \alpha_s^n \tag{14}$$

6.1. Renormalons and gluon condensate

According to the usual wisdom, the coefficient c_n of series is expected to grow like $n!$ asymptotically (Infrared Renormalon), which is expected to be absorbed into the perturbative part of the gluon condensate $\langle\alpha_s G^2\rangle$, while the uncertainty of the asymptotic series would induce a term proportionnal to Λ^4 which is the *physical* gluon condensate appearing in the OPE. Moreover, the full value of the gluon condensate can be measured with high-accuracy from the lattice as it is the plaquette action. [4] It can be expressed as :

$$\Delta P_N \equiv P_{exact} - \sum_N p_n \alpha_s^n \approx (\Lambda \cdot a)^{\rho_N} \tag{15}$$

where P_{exact} is the exact plaquette action, p_n is the perturbative coefficient calculated explicitly and a is the lattice spacing. Fitting the lattice data on the difference ΔP_N [79] using power-like corrections, one finds:

$$\rho_N \simeq \begin{array}{ll} 2 & for \quad N \leq 10 \\ 4 & for \quad N \geq 10\,. \end{array} \tag{16}$$

The previous result indicates that numerically the PT series can differentiate between the quadratic and quartic corrections and shows some duality between long perturbative series and power corrections. Another remarkable feature from lattice calculations is the fact that:

$$r_n \equiv \frac{p_{n+1}}{p_n} \approx constant\,, \tag{17}$$

indicating that the series grow geometrically. Some other QCD processes present analogous properties [86]. These results indicate no alternate signs for the terms of the QCD PT series, which then do not support (to the order where the series are evaluated) the presence of the Ultraviolet Renormalon. They also indicate that the PT series do not show any existence of Infrared Renormalon (no $n!$ growth of the PT series) at that order. All these features can change our dogma on the understanding of higher order perturbative series.

6.2. Quadratic corrections

A similar observation of duality between the long PT series and quadratic corrections has been reported in the analysis of the Coulomb potential where the quadratic correction at short distance:

$$V_{\bar{Q}Q}|_{non-pert} = \sigma \cdot R\,, \tag{18}$$

(R is the distance between two heavy quarks and σ is the string tension) is reproduced by higher order PT series [80]. Another similar decrease of the strength of the quadratic correction when adding higher order PT terms has been also noted in the neutrino DIS analysis of the xF_3 sum rule [81], where in this process, the quadratic corrections are instead associated to long distances [5].

What seems to emerge from the previous observations is the Duality between Long perturbative series and Quadratic corrections: one can use one of them but not both in order to avoid a double counting.

However, as there is no gauge invariant operator of dimension 2 in a field theoretical approach, it is difficult to parametrize analytically a such quadratic term. In fact, from the previous discussion, the quadratic term is a part

[4]Some pioneer attempt to measure the gluon condnesate on the lattice can be found in [78].

[5]Quadratic corrections also to play a rôle in the Regge trajectories of mesons [82].

Table 1: Estimate of $d_2 \equiv -(\alpha_s/\pi)\lambda^2$ in GeV$^2 \times 10^2$ from light quarks

Channels	d_2
e^+e^- data	
Ratio of moments $\mathcal{R}(\tau)$	6.5 ± 0.5
$\pi-$ sum rule	
Laplace $\mathcal{L}(\tau)$	12 ± 6
Lattice data	
Scalar+Pseudoscalar	$\simeq 12$
τ-decay	
$\Delta_N \equiv$ *large* $\beta - \sum_1^4$ PT series :	
Fixed Order PT	2.6 ± 0.8
Contour Improved	5.9 ± 0.8
Arithmetic Average	**7 ± 3**

of the Wilson coefficient of the unit operator which is difficult (in practice) to disentangle from the other terms of the PT series poorly known to $N \leq 3 \sim 4$. Some possible issue for explaining the origin of the quadratic term is the dual string model or the AdS/QCD approach [4].

7. Phenomenology of Quadratic corrections

7.1. Short-distance gluon mass

As emphasized in previous section, it is difficult to parametrize analytically the contribution of the quadratic correction despite its evidence from numerical simulations. An attempt to include this contribution in a gauge invariant way (to leading order in α_s) is to introduce a tachyonic gluon mass through the gluon propagator [83]:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2+\lambda^2} \tag{19}$$

A systematic evaluation of this contribution has been presented in [83] for different two-point correlators.

7.2. Correlated estimate of λ^2

The value of λ^2 has been fitted [6] from $e^+e^- \rightarrow$ *hadrons* data [30, 83], π-Laplace sum rule [83], and lattice data for the sum of pseudoscalar $\oplus$ scalar two-point correlators [85], where some eventual instanton contributions cancel out. We show the results in Table 1. One can note that the one from τ-decay [35] has been estimated from the difference between the large β-limit prediction and the sum of the known PT series which then depends on how the PT series is resummed (fixed order or contour improved).

[6]More detailed discussions can be found in [35, 84].

7.3. Effects of λ^2 on the light quark masses

The sum rule scale of the π-channel has been puzzling in the conventional approach where we have the hierarchy [65]:

$$M_\rho^2 \approx 0.6\,\text{GeV}^2 \ll M_\pi^2 \approx 2.7\,\text{GeV}^2\,, \tag{20}$$

while in the presence of λ^2:

$$M_\rho^2 \approx M_\pi^2 \approx 1.7\,\text{GeV}^2\,, \tag{21}$$

where the duality between the two sides of the π-sum rule improves. λ^2 decreases the estimate of the light quark masses by 5% from the (pseudo)scalar sum rules [83]. The extractions of the strange quark masses from e^+e^- and τ-decay data including the λ^2 corrections have been done in [35, 87]. The averages of the results from different forms of the sum rules are in MeV:

$$\begin{aligned} \overline{m}_u(2) &= 2.8(2)\,,\quad \overline{m}_d(2) = 5.1(2)\,, \\ \overline{m}_s(2) &= 96.1(4.8)\,. \end{aligned} \tag{22}$$

where the running masses have been evaluated at 2 GeV.

Table 2: Different QCD corrections to the τ hadronic widths.

Corrections	Size $\times 10^3$
$\delta_{svz} = \sum_4^8 \delta^{(D)}$	$-(7.8 \pm 1.0)$
$\delta_{st} \equiv \delta_{svz} + \delta_m^{(2)} + \delta_\pi + \delta_{a_0}$	$-(10.9 \pm 1.1)$
δ_{inst}	$-(0.7 \pm 2.7)/20$
δ_{DV}	$-(15 \pm 9)$
$\delta_2 \equiv$ *large* $\beta - \sum_1^4 PT$	(17 ± 5) FO
	(39 ± 5) CI
$\delta_{nst} \equiv \delta_{inst} + \delta_{DV} + \delta_2$	(2.0 ± 9.4) *FO*
	(24.0 ± 10.6) *CI*

7.4. Effects of λ^2 on α_s from τ-decays

The $\tau \rightarrow \nu_\tau +$ *hadrons* process is a good laboratory for testing the effects of these tiny deviations (duality violation and tachyonic gluon mass) from the conventional sum rules approach. The non-strange $\Delta S = 0$ component of the τ-hadronic width normalized to the electronic width can be expressed as:

$$\begin{aligned} R_\tau &= 3|V_{ud}|^2 S_{EW} \times \\ &\quad (1 + \delta^{(0)} + \delta'_{EW} + \delta_m^{(2)} + \delta_{\rm svz} + \delta_{\rm nst})\,, \end{aligned} \tag{23}$$

where $|V_{ud}| = 0.97418 \pm 0.00027$ [88] is the CKM mixing angle; $S_{EW} = 1.0198 \pm 0.0006$ [89] and $\delta'_{EW} = 0.001$ [90]. are known electroweak corrections; $\delta^{(0)}$ and $\delta_m^{(2)}$ are the perturbative and light quark mass corrections; $\delta_{\rm svz} \equiv \sum_{D=4}^8 \delta^{(D)}$, is the sum of the non-perturbative (NP) contributions of dimension D within the SVZ expansion [1], while $\delta_{\rm nst}$ are some eventual NP effects not

included into $\delta_{\rm svz}$, which are here due to instanton (δ_{inst}), quadratic corrections (δ_2) and duality violations (δ_{DV}). The size of different contributions are given in Table 2, where it is remarkable to note that the contributions of duality violation and quadratic corrections tend to cancel out. Using: $R_{\tau,V+A}|_{exp} = 3.479 \pm 0.011$, we deduce the result in Table 3, which we compare with some other determinations. For completing, the previous values of the QCD perturbative parameters, we give the recent values of the c, b running quark masses from ratio of moments to order α_s^3 [57] in units of MeV:

$$\overline{m}_c(m_c) = 1261(18)\,, \quad \overline{m}_b(m_b) = 4232(10)\,. \quad (24)$$

Table 3: Value of α_s rom τ-decay and comparison with the one from $Z \to hadrons$ [91] and the world average [88, 92]

PT	$\alpha_s(M_\tau)$	$\alpha_s(M_Z)$
FO	$0.3276\ (34)_{\rm ex}(86)_{\rm th}$	$0.1195\ (4)_{\rm ex}(10)_{\rm th}(2)_{\rm ev}$
CI	$0.3221\ (48)_{\rm ex}(122)_{\rm th}$	$0.1188\ (6)_{\rm ex}(15)_{\rm th}(2)_{\rm ev}$
$\langle\ \rangle$	$0.3249\ (29)_{\rm ex}(75)_{\rm th}$	$0.1192\ (4)_{\rm ex}(9)_{\rm th}(2)_{\rm ev}$
Z		$0.1191\ (27)_{\rm ex}(2)_{\rm th}$
$\langle\ \rangle$		0.1184 (7)

We also show in Table 4 the values of non-pertubative condensates determined from QSSR.

Table 4: Values of the QCD condensates from QSSR.

Condensates	Values [GeV]^d	Sources
$\langle\bar{u}u\rangle(2)$	$-(0.254 \pm .015)^3$	(pseudo)scal,
$\langle\bar{d}d\rangle/\langle\bar{u}u\rangle$	$1 - 9\times 10^{-3}$	non-norm. ord. (pseudo)scal,
$\langle\bar{s}s\rangle/\langle\bar{d}d\rangle$	0.74(3)	non-norm. ord. (pseudo)scal
		$\oplus$ light & heavy baryons
$\langle\alpha_s G^2\rangle$	$7(2)10^{-2}$	e^+e^-, $\Upsilon - \eta_b$, J/ψ Laplace
		τ, J/ψ mom : unconclusive
$g\langle\bar{\psi}G\psi\rangle$	$M_0^2 = 0.80(2)$	Light baryons, B, B^*
$g^3\langle G^3\rangle$	$(31 \pm 13)\langle\alpha_s G^2\rangle$	J/ψ-mom
$\rho\alpha_s\langle\bar{\psi}\psi\rangle^2$	$(4.5 \pm 0.3)10^{-4}$	$\Longrightarrow \rho = 2.1 \pm 0.2$

Conclusions

We have presented in a compact form the developments of the SVZ sum rules 30 $\oplus$ 1 years later and have summarized in different Tables the values of the QCD parameters from the approach. Its applications are rich and have the advantage (compared to lattice simulations) to be analytical. The successful applications of the sum rules motivate continuous phenomenological applications, and improvements of the approach in connection with its new developments discussed here and by SZ in this jubilee, namely the inclusion of radiative and quadratic corrections, the study of duality violation and the connection of QSSR to dual AdS/QCD models.

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